

# Quantum Structure of Flat Spacetime and Schwarzschild Mass Uncertainty

Aharon Davidson\* and Ben Yellin†

*Physics Department, Ben-Gurion University of the Negev, Beer-Sheva 84105, Israel*

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Generalizing Dirac's procedure to time dependent constrained systems, we derive a reduced total Hamiltonian, resembling an upside down harmonic oscillator, which generates the Schwarzschild solution in the mini superspace. Associated with the circumferential radius dependent Schrodinger equation is a tower of localized Guth-Pi-Barton wave packets, orthonormal and non-singular, admitting equally spaced average-'energy' levels. Our approach is characterized by a universal quantum mechanical uncertainty structure which enters the game already at the flat spacetime level, and accompanies the massive Schwarzschild sector for any arbitrary mean mass. The average black hole horizon area is linearly quantized.

In the beginning Newton postulated a universal gravitational force law exerted by a massive point particle on bodies floating in a flat background space and sharing an invariant ticking time. Special relativity, while unifying space and time, has not challenged the flatness of the resulting spacetime nor its non-dynamical role. The general relativity revolution elevated spacetime into a dynamical object, albeit classical, with Newton's law embedded within the celebrated yet singular Schwarzschild solution. The next challenge is to reveal how would the concept of spacetime be revised when quantum mechanics enters the game, and how would Newton's force law, and in particular the black hole event horizon fit in. Given the fact that quantum gravity, with its anticipated Planck scale effects, is still absent, the conventional assumption is that probing the structure of spacetime is premature at this stage. In this respect, however, the practical lesson to be deduced from Hawking-Bekenstein [1] black hole thermodynamics is that some combined gravitational and quantum mechanical effects may be currently attainable after all. In this paper, generalizing Dirac's procedure for circumferential radius dependent constrained systems, we quantize the Schwarzschild black hole solution (actually the static spherically symmetric geometry). We do it within the framework of the mini superspace [2], with the aspiration, however, to shed some light on the quantum mechanical structure of spacetime in general. The notion of a source point particle is replaced by a localized orthonormal Guth-Pi-Barton [3] wave packet, and the universal quantum mechanical mass uncertainty structure which governs the Schwarzschild vacuum/massive black hole is revealed.

Black hole thermodynamics is anchored to the area entropy formula [1]. The latter points towards some kind of microphysical degrees of freedom, but does not tell us what they are, where they live, and how to count them. In the absence of a satisfactory general relativistic quantum mechanical answers, it has been argued that the resolution must lie beyond general relativity, in string theory to be precise. Truly, stringy black holes have been proven valuable in this respect, supporting the holographic principle [4], and providing at least a partial answer [5] in

terms of D-branes. They shed light on the general issue, on extremal black hole in particular, but unfortunately, not directly on the (say) Schwarzschild black hole. In this paper, however, we do without relying on string theory or loop quantum gravity. Our approach differs from previous approaches (for a partial list, see Ref.[6]) by the universal quantum mechanical structure which forcefully enters already at the flat spacetime level. For simplicity, we adopt the Planck units  $c = G = \hbar = k_B = 1$ .

Let our starting point be the static spherically symmetric line element

$$ds^2 = -T(r)dt^2 + \frac{dr^2}{R(r)} + S^2(r)d\Omega^2. \quad (1)$$

A gauge fixing option is still at our disposal, but as far as the forthcoming Hamiltonian formalism is concerned, we have to exercise it with extra caution. The more so at the mini superspace level, where the general relativistic action  $\int \mathcal{R}\sqrt{-g} d^4x$  is integrated out over time and solid angle into  $\int \mathcal{L}dr$ . After integrating out the second derivative surface terms we are left with

$$\mathcal{L} = - \left( 1 + RS'^2 + \frac{RT'SS'}{T} \right) \sqrt{\frac{T}{R}}, \quad (2)$$

where  $T(r), R(r), S(r)$  serve as canonical variables.

The non-dynamical  $R(r)$  is the analogous lapse function. It is well known that pre-fixing  $R(r)$ , say  $R(r) = 1$ , is problematic. At the classical level, the algebraic Hamiltonian constraint which in turn adds a superfluous degree of freedom to the Schwarzschild solution is gone. Once  $R(r)$  is elevated to the level of an essential canonical variable, the question is whether the canonical role of  $T(r), S(r)$  can be relaxed? In other words, having in mind canonical quantum gravity [7], can one harmlessly pre-fix one of them before conducting the variation?

The answer to this question is in the affirmative, and can be supported by a simple example. By pre-fixing the circumferential radius  $S(r) = r$ , one can easily verify that the residual Lagrangian

$$\mathcal{L} = (rR' + R - 1) \sqrt{\frac{T}{R}} \quad (3)$$

does produce the exact Schwarzschild solution and nothing else. In the mini superspace Lagrangian formalism, the coordinate  $r$  can still be redefined  $r \rightarrow f(r)$  (involving an explicit function of  $r$ ), but does not take any part whatsoever in the variational process itself. It is thus advantageous, and in some respect even necessary, to use a pre-gauge which is capable of constituting an invariant geometrical quantity which is furthermore canonical variable independent. For the hereby adopted  $S(r) = r$  gauge, it is the invariant spherical surface area  $A(r) = 4\pi r^2$  which is  $T, R$ -independent as required. The procedure has been successfully implemented in the cosmological case [8], where an admissible gauge choice fixes not the lapse function, but rather the scale factor.

The path to the quantum black hole is governed by the Hamiltonian formalism, and to be more specific, by Dirac's prescription [9] for dealing with constraint systems. Given the Lagrangian eq.(3), and borrowing the language of analytical mechanics, the corresponding momenta  $p_R = \frac{\partial \mathcal{L}}{\partial R'}$  and  $p_T = \frac{\partial \mathcal{L}}{\partial T'}$  fail to determine the velocities  $R'$  and  $T'$ . This, in turn, gives rise to the two primary constraints

$$\phi_1 = p_R - r\sqrt{\frac{T}{R}} \approx 0, \quad \phi_2 = p_T \approx 0. \quad (4)$$

The fact that their Poisson brackets does not vanish

$$\{\phi_1, \phi_2\} = -\frac{r}{2\sqrt{TR}} \neq 0, \quad (5)$$

makes them second class constraints. As argued by Dirac, the naive Hamiltonian  $\mathcal{H}_{naive} = p_R R' + p_T T' - \mathcal{L}$  is not uniquely determined, and one may add to it any linear combination of the  $\phi$ 's, which are zero, and go over to  $\mathcal{H}^* = \mathcal{H}_{naive} + \sum_i u_i \phi_i$ . Consistency then requires the constraints be constants of motion, and as such, they must weakly obey

$$\frac{d\phi_i}{dr} = \{\phi_i, \mathcal{H}_{naive}\} + \sum_j u_j \{\phi_i, \phi_j\} + \frac{\partial \phi_i}{\partial r} \approx 0. \quad (6)$$

Calculating the various Poisson brackets, we solve these linear equations to find out that

$$u_R = \frac{1-R}{r}, \quad u_T = \frac{T(1-R)}{rR}. \quad (7)$$

Finally, substituting the coefficients  $u_{R,T}$  into  $\mathcal{H}^*$  constitutes the so-called total Hamiltonian

$$\mathcal{H}_{total} = (1-R) \left( \sqrt{\frac{T}{R}} + \frac{1}{r} \left( p_R - r\sqrt{\frac{T}{R}} \right) + \frac{T p_T}{rR} \right). \quad (8)$$

A generalization of Dirac's prescription for time dependent ( $r$ -dependent in our case) constrained Hamiltonians [10] is in order. One is quite familiar with the Dirac brackets technique, invoked to make the entire set

of constraints first class, but in the presence of explicit time dependence (in the Hamiltonian and/or in the constraints themselves), an extra step must be taken. And indeed, the constraints leave their impact on the equations of motion via the  $r$ -evolution operator formula

$$\frac{d}{dr} = [\cdot, \mathcal{H}_{total}]_D + \frac{\partial}{\partial r} \Big|_D, \quad (9)$$

where

$$[X, Y]_D \equiv \{X, Y\} + \frac{\epsilon_{ij}}{\{\phi_1, \phi_2\}} \{X, \phi_i\} \{\phi_j, Y\}, \quad (10)$$

$$\frac{\partial X}{\partial r} \Big|_D \equiv \frac{\partial X}{\partial r} + \frac{\epsilon_{ij}}{\{\phi_1, \phi_2\}} \{X, \phi_i\} \frac{\partial \phi_j}{\partial r}. \quad (11)$$

As consistency checks we have verified that all Dirac brackets involving the  $\phi_{1,2}$  constraints vanish, and so do the dressed partial derivatives  $\frac{\partial \phi_i}{\partial r} \Big|_D$ , and have reassured the emergence of the classical Schwarzschild solution. Among the non-vanishing Dirac brackets we pick up to present the conventional  $[R, p_R]_D = 1$ , accompanied by the unconventional

$$[R, T]_D = \frac{2\sqrt{TR}}{r}, \quad (12)$$

which are both of relevance for our forthcoming discussion. Eq.(12) comes with a message; it simply tells us that the Schwarzschild metric components  $T, R$  would not commute when elevated to the level of quantum mechanical operators.

Explicitly imposing now the  $\phi_{1,2}$  constraints (thereby importing them to the quantum level), and subsequently substituting

$$T = \frac{p_R^2 R}{r^2}, \quad (13)$$

we are led to the reduced Hamiltonian

$$\mathcal{H}_{reduced} = \frac{1}{r}(1-R)p_R, \quad (14)$$

subject to the canonical Poisson brackets  $[R, p_R]_P = 1$ . The Schwarzschild solution  $R = 1 - 2m/r$ ,  $p_R = kr$  (choosing  $k$  is nothing but rescaling  $t$ ) is then straight forwardly confirmed. The physical role played by the momentum  $p_R$  is manifest via eq.(13) which serves now as the connection with the underlying metric.

A radial marker redefinition  $r = e^\rho$  then transforms the  $r$ -dependent reduced Hamiltonian eq.(14) into an  $\rho$ -independent variant of the  $xp$ -type [11] discussed in the context of Riemann zeta function zeroes. A successive linear canonical transformation

$$1-R = \frac{1}{\sqrt{2}}(p-x), \quad p_R = \frac{1}{\sqrt{2}}(p+x), \quad (15)$$

yields the upside-down harmonic oscillator Hamiltonian

$$\mathcal{H} = \frac{1}{2} (p^2 - x^2) . \quad (16)$$

Combined with the latter Hamiltonian is the  $\rho$ -dependent Schrodinger equation

$$-\frac{\partial^2 \psi}{\partial x^2} - x^2 \psi = 2i \frac{\partial \psi}{\partial \rho} . \quad (17)$$

The 'energy' eigenfunctions, proportional to the Hermite polynomials  $\psi_E \sim e^{\mp \frac{i x^2}{2}} H(-\frac{1}{2} \mp i E, \pm e^{\pm \frac{i \pi}{4}} x) e^{-i E \rho}$ , pose a major problem. Owing to their  $1/\sqrt{|x|}$  behaviors at  $x \rightarrow \pm\infty$ , they are not square integrable. Counter intuitively, however, especially when dealing with an unbounded potential, there exists a set of localized wave packets which satisfy the above  $\rho$ -dependent Schrodinger equation.

### The vacuum case

The basic wave packets are of the generic form

$$\psi_n(x, \rho) = P_n(x, \rho) e^{-\frac{x^2}{2} \tan(\varphi - i\rho)} , \quad (18)$$

where  $P_n(x, \rho, \varphi) = \sum_{k=0}^n c_k(\rho, \varphi) x^k$  are even/odd polynomials of order  $n$ . Note that the differential equation

$$\frac{\partial^2 P}{\partial x^2} - \tan(\varphi - i\rho) \left( P + 2x \frac{\partial P}{\partial x} \right) + 2i \frac{\partial P}{\partial \rho} = 0 \quad (19)$$

does allow for a more general series expansion, namely  $P = c_n x^n + c_{n-2} x^{n-2} + \dots$ , but unless  $n$  is an integer, the series does not terminate, turning the solution singular. The wave packet solution eq.(34) is characterized by a real parameter  $\varphi$  which controls the  $\rho$ -dependent width of the wave packet

$$\delta(\rho, \varphi) = \left( \frac{\cos 2\varphi + \cosh 2\rho}{2 \sin 2\varphi} \right)^{1/2} . \quad (20)$$

The condition  $\sin 2\varphi > 0$  then suffices to assure the tenable behavior  $\psi_n \rightarrow 0$  as  $x \rightarrow \pm\infty$ . Altogether, being non singular [12], square integrable, and furthermore orthonormal  $\int_{-\infty}^{\infty} \psi_n^\dagger \psi_m dx = \delta_{nm}$ , these wave packets pass all fundamental physical requirements. The first three normalized wave packets on the list are given explicitly by

$$\psi_0 = \frac{\sin^{\frac{1}{4}} 2\varphi e^{-\frac{x^2}{2} \tan(\varphi - i\rho)}}{(2\pi)^{\frac{1}{4}} \cosh^{\frac{1}{2}}(\rho + i\varphi)} , \quad (21)$$

$$\psi_1 = \frac{\sin^{\frac{3}{4}} 2\varphi x e^{-\frac{x^2}{2} \tan(\varphi - i\rho)}}{(2\pi)^{\frac{1}{4}} \cosh^{\frac{3}{2}}(\rho + i\varphi)} , \quad (22)$$

$$\psi_2 = \frac{\sin^{\frac{5}{4}} 2\varphi (x^2 - \delta^2(\rho, \varphi)) e^{-\frac{x^2}{2} \tan(\varphi - i\rho)}}{\sqrt{2}(2\pi)^{\frac{1}{4}} \cosh^{\frac{5}{2}}(\rho + i\varphi)} . \quad (23)$$

Notice that, reflecting their non-trivial  $\rho$ -dependence, the polynomials involved are not the Hermite polynomials.

The ground state  $\psi_0$  has been introduced by Guth-Pi and Barton [3], with  $t$  replacing  $\rho$  of course, when discussing the quantum mechanics of the scalar field in the so-called new inflationary universe. The raising and lowering operators are given by

$$b^\pm = \frac{\pm i}{\sqrt{\sin 2\varphi}} (p \cosh(\rho \mp i\varphi) - x \sinh(\rho \mp i\varphi)) \quad (24)$$

$$b^- \psi_n = (-1)^n \sqrt{n} \psi_{n-1} , \quad (25)$$

$$b^+ \psi_n = (-1)^{n+1} \sqrt{n+1} \psi_{n+1} , \quad (26)$$

giving the Hamiltonian the form

$$\mathcal{H} = -\frac{b^{+2} + b^{-2} + \cos 2\varphi (b^+ b^- + b^- b^+)}{2 \sin 2\varphi} . \quad (27)$$

Owing to  $\langle b^{\pm 2} \rangle = 0$ , associated with the wave packets are then the global, meaning  $\rho$ -independent, average-'energy' levels

$$E_n = \int_{-\infty}^{\infty} \psi_n^\dagger \mathcal{H} \psi_n dx = -\left(n + \frac{1}{2}\right) \cot 2\varphi . \quad (28)$$

The choice  $\cot 2\varphi < 0$  gives rise to a positive spectrum, and combining with the previous integrability condition  $\sin 2\varphi > 0$ , the still arbitrary angle  $\varphi$  gets restricted to the region  $\frac{\pi}{4} < \varphi < \frac{\pi}{2}$ .

Constructing the set of localized wave packets, we can now calculate quantum mechanical expectation values associated with the various metric component operators. To do so, we first notice a direct consequence of the discrete symmetry  $x \rightarrow -x$ , namely  $\langle x \rangle_n = \langle p \rangle_n = 0$ . And then, recalling the relations eq.(15), we find

$$\langle 1 - R \rangle_n = 0 , \quad \langle (1 - R)^2 \rangle_n = \frac{(2n+1)e^{-2\rho}}{2 \sin 2\varphi} , \quad (29)$$

$$\langle p_R \rangle_n = 0 , \quad \langle p_R^2 \rangle_n = \frac{(2n+1)e^{2\rho}}{2 \sin 2\varphi} , \quad (30)$$

with the associated uncertainty relation reading

$$\Delta R \Delta p_R = \frac{2n+1}{2 \sin 2\varphi} \geq \frac{1}{2} , \quad (31)$$

in accord with the Dirac brackets eq.(12). These formulas allow us to make contact with the familiar general relativistic Schwarzschild solution  $1 - R = 2m/r$  and  $T/R = p_R^2/r^2 = \text{const}$  (the constant can always be absorbed by a time rescaling), with the dictionary reading  $r = e^\rho$ . We refer to the emerging spacetime as the quantum mechanical Schwarzschild vacuum. It is massless on average, but exhibits a non-zero mass uncertainty

$$\langle m \rangle_n \pm \Delta m_n = 0 \pm \sqrt{\frac{2n+1}{8 \sin 2\varphi}} , \quad (32)$$

which can be interpreted an equal amount of positive and negative mass metric fluctuations. Note that the

quantum uncertainty is bounded from below  $\Delta m \geq \frac{1}{2\sqrt{2}}$  and cannot disappear.

One may wonder though what goes wrong when attempting to construct an eigenvacuum  $\Psi$ , namely a zero eigenmass state of the mass operator  $\hat{m} \sim e^{i\varphi} b^- + e^{-i\varphi} b^+$  (see the forthcoming eq.(42)). It takes some algebra to prove that  $\Psi \sim \sum_k c_k \psi_{2k}$ , with the coefficients subject to the series expansion

$$(1 - y^2)^{-1/2} = \sum_k |c_k|^2 y^{2k}. \quad (33)$$

The sum  $\sum_k |c_k|^2$  diverges, giving rise to unacceptable quantum mechanical consequences.

### The massive case

The inclusion of mass requires the violation of the discrete  $x \rightarrow -x$  symmetry of the vacuum wave function. This is done by simply shifting the Gaussian of the Guth-Pi-Barton tower, with the elaborated wave functions taking the form

$$\psi_n(x, \rho) = \tilde{P}_n(x, \rho) e^{-\frac{\eta}{2} \tan(\varphi - i\rho) - \eta x \sec(\varphi - i\rho)}, \quad (34)$$

introducing the shift parameter  $\eta$  designated to induce the mean Schwarzschild mass. The modified polynomials  $\tilde{P}_n(x, \rho)$  generalize the previous  $P_n(x, \rho)$ , and contain now even as well as odd powers of  $x$ . The first normalized wave packets are given explicitly by

$$\psi_0 = \frac{\sin^{\frac{1}{4}} 2\varphi e^{-\frac{1}{2}(x^2 + \eta^2) \tan(\varphi - i\rho) - \eta x \sec(\varphi - i\rho)}}{(2\pi)^{\frac{1}{4}} e^{\frac{1}{2}\eta^2 \cot \varphi} \cosh^{\frac{1}{2}(\rho + i\varphi)}} \quad (35)$$

$$\psi_1 = \left( \eta \cot \phi + \frac{x - i\eta \sinh(\rho + i\varphi)}{\cosh(\rho + i\varphi)} \right) \frac{\sin^{\frac{3}{4}} 2\varphi e^{-\frac{1}{2}(x^2 + \eta^2) \tan(\varphi - i\rho) - \eta x \sec(\varphi - i\rho)}}{(2\pi)^{\frac{1}{4}} e^{\frac{1}{2}\eta^2 \cot \varphi} \cosh^{\frac{1}{2}(\rho + i\varphi)}}. \quad (36)$$

The raising and lowering operators get shifted

$$b_\eta^\pm = b^\pm - \frac{\eta}{\sqrt{\sin 2\varphi}}, \quad (37)$$

in obvious notations. The Hamiltonian in the new basis resembles eq.(27), with  $b_\eta^\pm$  replacing  $b^\pm$ , but gets further supplemented by

$$-\frac{\eta \cot \varphi}{\sqrt{\sin 2\varphi}} (b_\eta^+ + b_\eta^-) - \frac{\eta^2}{2 \sin^2 \varphi} \quad (38)$$

Owing to  $\langle b_\eta^\pm \rangle = 0$ , associated with the new set is again a ladder average-'energy' spectrum, but it is now uniformly shifted relative to the vacuum ladder. To be specific,

$$E_n = -\left(n + \frac{1}{2}\right) \cot 2\varphi - \frac{\eta^2}{2 \sin^2 \varphi}. \quad (39)$$

Had we adopted the special values  $\eta_\ell^2 = \ell(\tan \varphi - \sin 2\varphi)$  ( $\ell$  integer) we would have in fact recaptured the vacuum average-'energy' ladder

$$E_{n,\ell} = -\left(n - \ell + \frac{1}{2}\right) \cot 2\varphi. \quad (40)$$

The various massive wave packets are characterized by the  $n$ -independent quantum averages

$$\langle x \rangle_n = -\frac{\eta \cosh \rho}{\sin \varphi}, \quad \langle p \rangle_n = -\frac{\eta \sinh \rho}{\sin \varphi}, \quad (41)$$

and hence share the one and the same classical Schwarzschild metric, with  $\frac{1}{\sqrt{2}} \langle p - x \rangle = 2\langle m \rangle / r$ . The mass operator itself can be expressed in terms of the raising and lowering operators

$$\hat{m} = \frac{1}{2\sqrt{2}} \left( \frac{\eta}{\sin \varphi} + \frac{e^{i\varphi} b^- + e^{-i\varphi} b^+}{\sqrt{\sin 2\varphi}} \right). \quad (42)$$

The two uncertainties  $\Delta R$  and  $\Delta p_R$  turn out to be insensitive to the presence of the  $\eta$ -parameter, retaining the exact vacuum value, with eq.(31) untouched. Altogether, associated with the quantum mechanical Schwarzschild black hole of the  $n$ -th state is the mass formula

$$\langle m \rangle_n \pm \Delta m_n = \frac{\eta}{2\sqrt{2} \sin \varphi} \pm \sqrt{\frac{2n+1}{8 \sin 2\varphi}}. \quad (43)$$

Several remarks are in order:

- (i) The sign of  $\eta$  is as arbitrary as the sign of the mass parameter in the original Schwarzschild solution.
- (ii) The larger  $\eta$  is, the more negligible is the  $\frac{\Delta m}{m}$  ratio, driving the solution into a more classical regime.
- (iii) The larger is  $\eta$ , the larger is the uniform shift downwards, see eq.(39), of the average-'every' levels. This in turn increases the number of the low 'energy' states which actually penetrate the upside-down harmonic potential barrier.
- (iv) While the underlying classical gravitational metric is governed by  $\langle 1 - R \rangle$ , it is independent of  $\langle p_R \rangle$  which can be absorbed by a time rescaling.

The wave functions  $\psi_n$  are not sharply peaked about any particular classical trajectory, and in particular, do not seem to exhibit any exceptional behavior at  $r = 2\langle m \rangle$  which marks the classical location of the black hole event horizon. However, the fact that the universal variance eq.(29) is kept unchanged in the massive sector may have important consequences for black hole thermodynamics. The central geometrical role here is played by the horizon surface area  $A$ . Classically, we know that  $A = 16\pi m^2$  for  $m > 0$ , but this leaves the door quantum mechanically open for the ambiguity  $\langle A \rangle \sim \langle m^2 \rangle$  versus  $\langle A \rangle \sim \langle m \rangle^2$ . To make a decision, we remind the reader that treating the horizon surface area as an adiabatic invariant, an equally spaced Bohr-Sommerfeld area spectrum has been conjectured by Bekenstein [13] and subsequently modelled by Bekenstein-Mukhanov [14]. In our case, while  $\langle m \rangle$  is  $n$ -independent, it is  $\langle m^2 \rangle$  which is linearly quantized as required, implying

$$\langle A \rangle_n = 16\pi \langle m^2 \rangle_n = 2\pi \left( \frac{\eta^2}{\sin^2 \varphi} + \frac{2n+1}{\sin 2\varphi} \right). \quad (44)$$



Representing the vacuum structure, the existence of a minimal surface area  $\langle A \rangle_{min} = 2\pi/\sin 2\varphi$  is noticeable, advocating the case of a universally fixed  $\varphi$ . It should be emphasized, however, that despite of the apparent similarity, eq.(44) differs from Bekenstein quantization. While any two distinct states labeled by  $n_1 \neq n_2$  are conventionally interpreted to be associated with two distinct black holes of masses  $m_1 \neq m_2$ , they are associated in our case with a common  $\langle m \rangle$ . A closer inspection reveals that Bekenstein's conjecture can be anchored to eq.(40), with the minimal massive horizon surface area is then  $\langle A \rangle_{\ell=1, n=0} = 2\pi \tan \varphi$ .

The discussion presented in this paper, while hopefully shedding some light on what to expect when letting quantum mechanics meet general relativity, leaves a bunch of question marks open, some of which seem fundamental. In particular, the statistical role of the average 'energy' ladder  $E_n$  is yet to be challenged by black hole thermodynamics. Also,  $\varphi$  is an arbitrary parameter at this stage, but there is a good reason to suspect that it is uniquely fixed. At any rate, looking at the half full glass, we have demonstrated that one can (i) Probe the vacuum/massive Schwarzschild black hole quantum mechanics even though quantum gravity is still absent, (ii) Reveal the universal quantum mechanical structure of the Schwarzschild black hole geometry, and contrary to the conventional wisdom (iii) Do it without appealing to theories beyond general relativity, such as string theory or loop gravity.

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\* Electronic address: davidson@bgu.ac.il; Homepage: <http://www.bgu.ac.il/~davidson>

† Electronic address: yellinb@bgu.ac.il

- [1] S.W. Hawking, Phys. Rev. Lett. 26, 1344 (1971); J.D. Bekenstein, Lett. Nuov. Cimento 4, 737 (1972); J.D. Bekenstein, Phys. Rev. D7, 2333 (1973); S.W. Hawking, Nature 248, 30 (1974); J.D. Bekenstein, Phys. Rev. D9, 3292 (1974); S.W. Hawking, Comm. Math. Phys. 43, 199 (1975).
- [2] B.S. DeWitt, Phys. Rev. 160, 1113 (1967); J.A. Wheeler, in *Battelle Rencontres*, 242 (Benjamin NY, 1968); W.E. Blyth and C. Isham, Phys. Rev. D11, 768 (1975); S.W. Hawking and I.G. Moss, Phys. Lett. 110B, 35 (1982); J. Hartle and S.W. Hawking, Phys. Rev. D28, 2960 (1983); J.J. Halliwell and S.W. Hawking, Phys. Rev. D31, 1777 (1985).
- [3] A.H. Guth and S.Y. Pi, Phys. Rev. D32, 1899 (1985); G. Barton, Ann. of Phys. 166, 322 (1986); A. Albrecht, P. Ferreira, M. Joyce and T. Prokopec, Phys. Rev. D50, 4807 (1994).
- [4] G. 't Hooft, in *Salam festschrift* A. Aly, J. Ellis, and S. Randjbar Daemi eds, (World Scientific, 1993), arXiv 9310026 [gr-qc]; L. Susskind, J. Math. Phys. 36, 6377 (1995); L. Susskind, Jour. Math. Phys. 36, 6377 (1995); D. Bigatti and L. Susskind, "Strings, branes and gravity" (Boulder), 883 (1999); R. Bousso, Rev. Mod. Phys. 74, 825 (2002).
- [5] C.G. Callen Jr., S.B. Giddings, J.A. Harvey and A. Strominger, Phys. Rev. D45, R1005 (1992); G.T. Horowitz and A. Strominger, Phys. Rev. Lett. 77, 2368 (1997); A. Strominger and C. Vafa, Phys. Lett. B379, 99 (1996); S.R. Das and S.D. Mathur, Annu. Rev. Nucl. Part. 50, 153 (2000); A. Sen, Gen. Rel. Grav. 40, 2249 (2008); M. Visser, JHEP 023, 1206 (2012).
- [6] K.V. Kuchar, Phys. Rev. D50, 3961 (1994); M. Cavaglia, V. de Alfaro and A.T. Filippov, Int. J. Mod. Phys. D4, 661 (1995), *ibid.* Int. J. Mod. Phys. D5, 227 (1996); J. F. Barbero G. and E.J.S. Villasenor, Living Rev. Rel. 13, 6 (2010).
- [7] R. Arnowitt, S. Deser, and C.W. Misner, Gen. Rel. Grav. 40, 1997 (2008).
- [8] A. Davidson and B. Yelin, arXiv:1206.0830v1 [gr-qc].
- [9] P.A.M. Dirac, in *Lectures on quantum mechanics*, (Dover publications, 1964).
- [10] A. Wipf, Lecture notes in Phys. 434, 22 (1994); M. de Leon, C. Marrero and D.M. de Diego, J. Phys. A29, 6843 (1996).
- [11] A. Connes, Selecta Math. 5, 29 (1999); M.V. Berry and J.P. Keating, SIAM Rev. 41, 236 (1999); G. Sierra, New Jour. Phys. 10, 033016 (2008); G. Sierra and P.K. Townsend, Phys. Rev. Lett. 101, 110201 (2008).
- [12] L. Modesto, Phys. Rev. D70, 124009 (2004); V. Husain and O. Winkler, Class. Quant. Grav. 22, L127 (2005).
- [13] J.D. Bekenstein, Lett. Nuovo Cim. 11, 467 (1974).
- [14] V. Mukhanov, Pis. Eksp. Teor. Fiz. 44, 50 (1986) [JETP Lett. 44, 63 (1986)]; J.D. Bekenstein and V.F. Mukhanov, Phys. Lett. B360, 7 (1995).